

Laws of motion

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Force

Force may be defined as an agency (external effect) which changes or tend to change the state of rest or of uniform motion

Force is vector quantity

SI unit is Newton (N). But in CGS it is Dyne

Inertia

The inherent property of a body by virtue of which it cannot change by itself its state of rest or of uniform motion in a straight line is called inertia.

Types of Inertia

- Inertia of rest
- Inertia of motion
- Inertia of direction

Mass as the measure of inertia, inertia means resistance to change.

Linear momentum: The amount of motion possessed by an object is the linear momentum. It is equal to the product of mass and velocity

$$P = mv \quad \text{kg m/s} \quad \text{kg m/s}$$

Newton's first law of motion

This law states that every body continues in its state of rest or uniform motion in straight line unless it is compelled by some external force to change that state.

Newton's second law of motion

It states that ~~are~~ the rate of change of momentum of a ~~&~~ body is directly proportional to the external force applied on the body and the change takes place in the direction of the applied force.

$p = mv$	Case I	Case II
By definition	$m = \text{constant}$ $v = \text{variable}$	$m = \text{variable}$ $v = \text{constant}$
$f = \frac{dp}{dt} = \frac{d}{dt}(mv)$	$f = m \left(\frac{dv}{dt} \right)$	$f = v \cdot \frac{dm}{dt}$
	$f = m \cdot a$	

Impulse :

Impulsive force :

A large force acting for a short time to produce a finite change in momentum is called an impulsive force

eg: force exerted by a bat while hitting a ball

Impulse of a force :

It is defined as the product of the force and the time for which it acts and is equal to the total change in momentum.

$$\boxed{\text{Impulse} = F \times t = \text{Total change in momentum}}$$

* Impulse is a vector quantity denoted by $\vec{J}$

* Its direction is along the direction of force. It may be zero, -ve or +ve.

Impulse momentum Theorem :

The impulse of a force = total change in momentum produced by the force

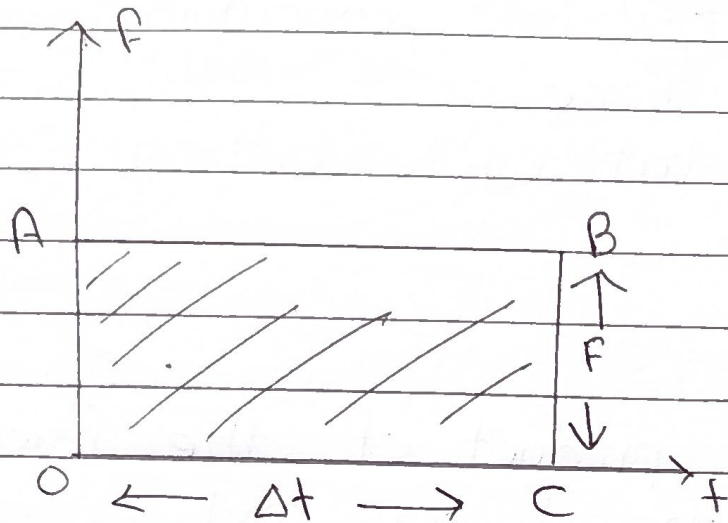
$$F = \frac{dp}{dt}$$

$$F dt = dp$$

$$\boxed{\int_0^t dt = \int_p^{pf} dp}$$

Measurement of impulse by graphical method

Case-I = When a constant force act on a body



Area = imp impulse

$$= OA \times OC$$

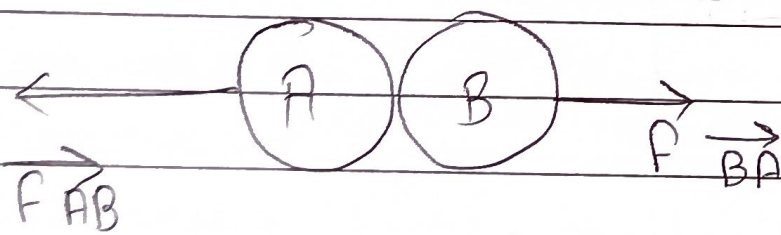
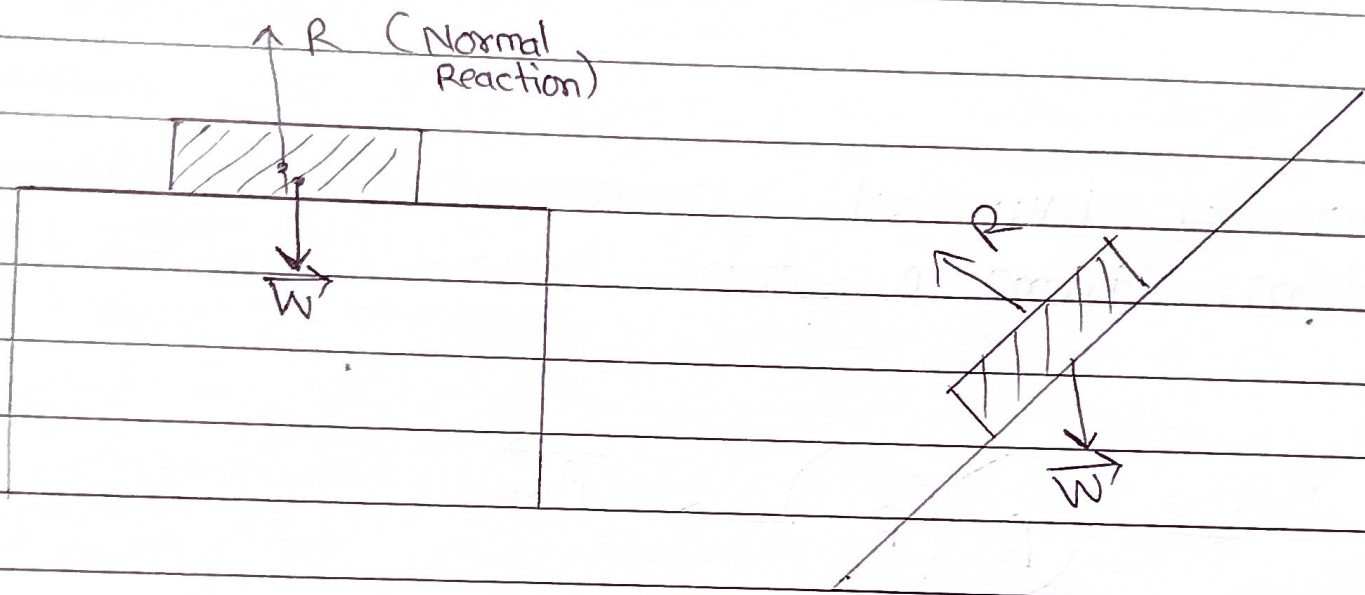
$$= F(\Delta t)$$

Case II

Newton's Third law of Motion.

"To every action, there is always an equal & opposite reaction."

Forces in nature always occur between pair of bodies. Force of body A by body B is equal and opposite to force on the body B by body A.



$$\vec{F}_{BA} = -\vec{F}_{AB}$$

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Conservation of linear momentum

If $F_{\text{ext}} = 0$

$$F = \frac{dp}{dt}$$

Hence, $p = \text{constant}$
 $mv = \text{constant}$

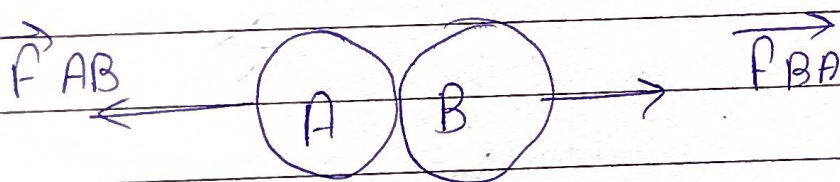
Then $p = \text{constant}$

$$\frac{dp}{dt} = 0$$

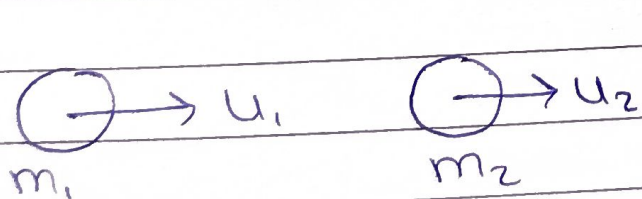
$$\therefore m \times \frac{1}{v}$$

$$\therefore dp = 0$$

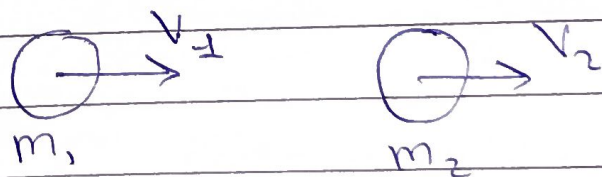
Derivation of law of conservation of linear momentum from Newton's third Law :-



$$\vec{F}_{AB} \Delta t = -\vec{F}_{BA} \Delta t$$



Before



After

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 (u_1 - v_1) = m_2 (u_2 - v_2)$$

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Practical Application of law of conservation of momentum. :-

Recoil of a Gun :-

Momentum of before firing = 0

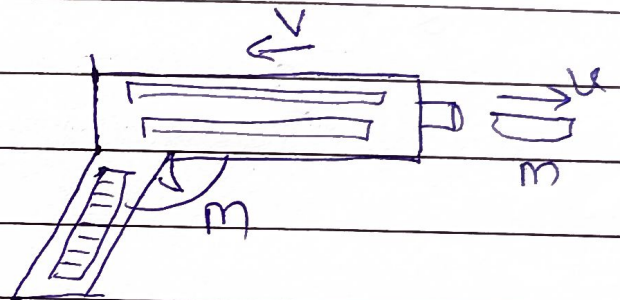
Momentum after firing = $mv + mv$

By law of conservation of momentum -

$$mv + mv = 0$$

$$mv = -mv$$

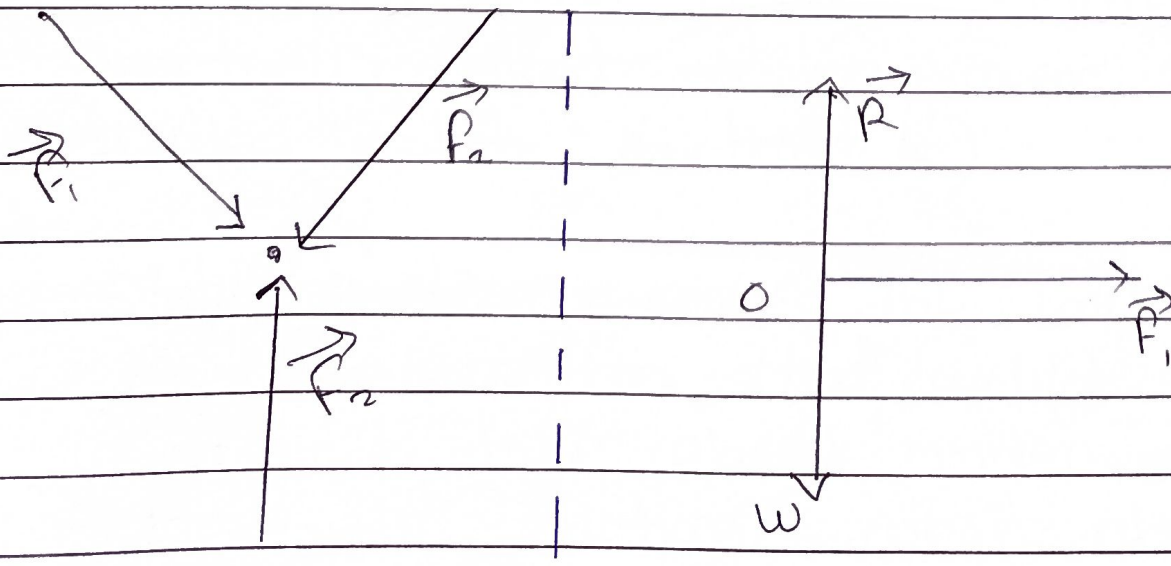
$$v = -\frac{m}{m} v$$



Equilibrium of concurrent forces

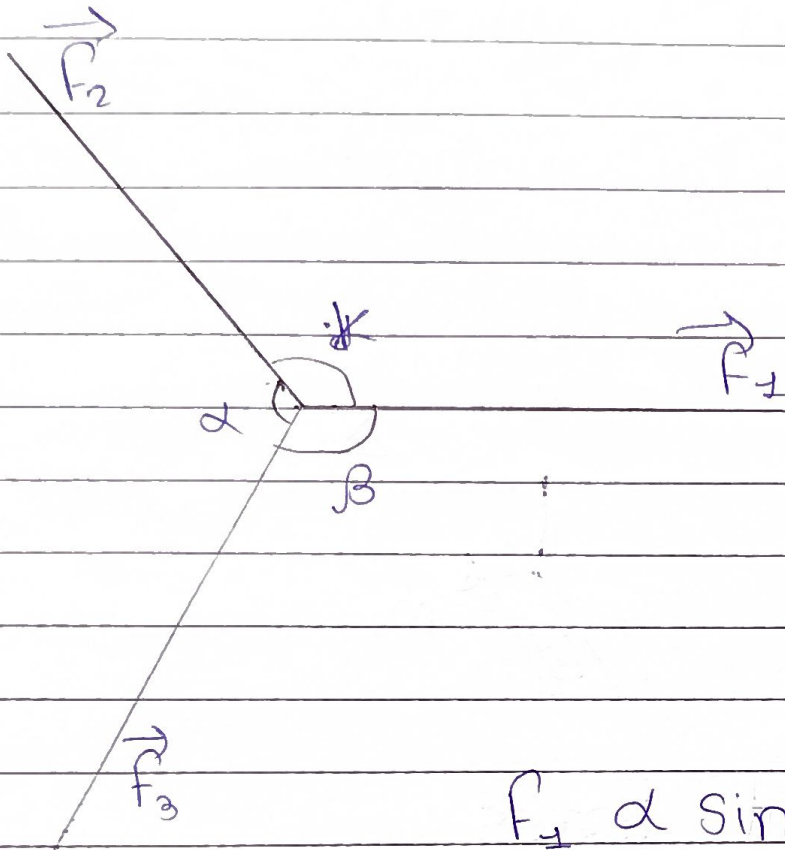
Force acting at the same point on a body are called concurrent forces.

When a no. of forces act on a body at the same point and the net unbalanced force is zero, the body will continue in its state of rest and is said to be in equilibrium.



Lami's Theorem:-

"If 3 forces acting on a particle kept it in a equilibrium, then each force is proportional to the sine of the angle between the other two forces."

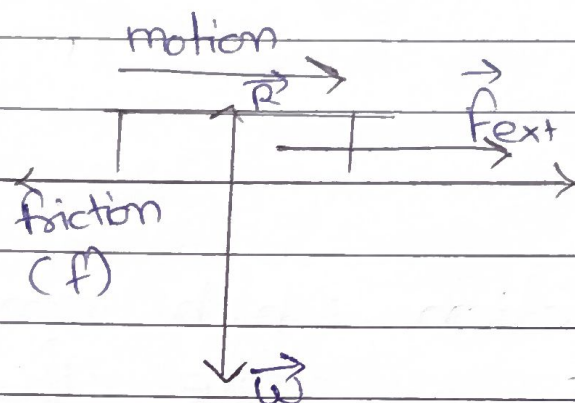


$F_1 \propto \sin \alpha$	$F_2 \propto \sin \beta$	$F_3 \propto \sin \gamma$
$\frac{F_1}{\sin \alpha} = \text{const}$	$\frac{F_2}{\sin \beta} = \text{constant}$	$\frac{F_3}{\sin \gamma} = \text{const}$

$$\boxed{\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}}$$

Friction

Whenever a body moves or tend to move over the surface of another body, a force comes into play which act parallel to the surface of constant contact and opposes the relative motion. This opposing force is called friction



Types of Friction :-

- 1) static friction
- 2) Limiting friction
- 3) kinetic friction

1) Static friction :

The force of friction which comes into play between two bodies before one body actually start moving over the other is called Static friction (f_s)

Static friction opposes the impending motion that is the motion that would take place (but does not take place) under the applied force, if friction were absent

Limiting friction :-

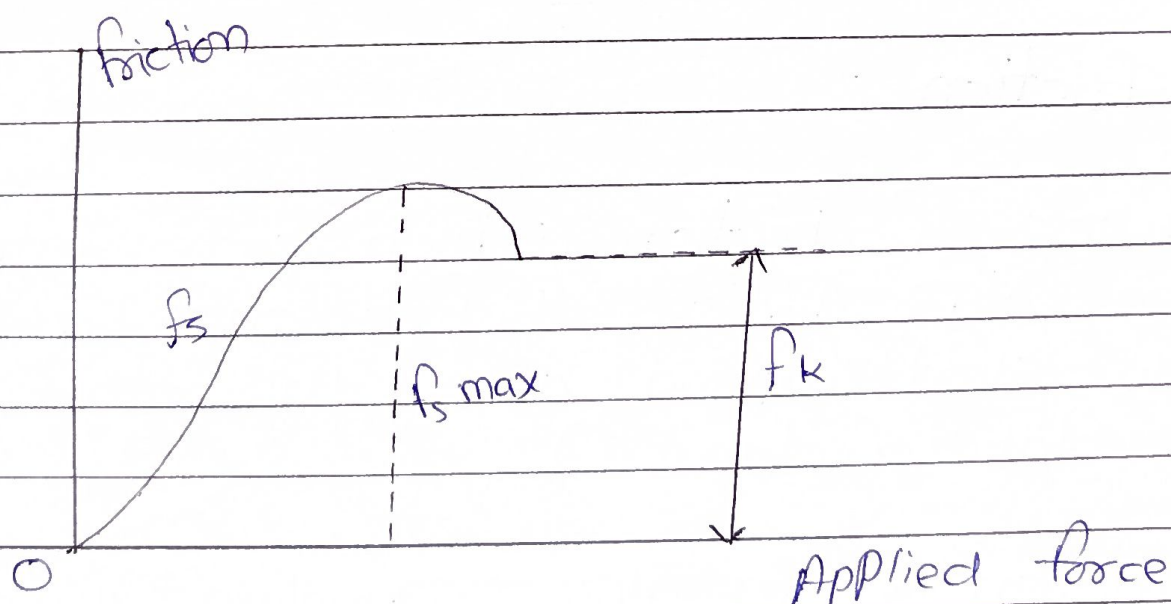
The maximum force of static friction ($f_s^{\max}$) which comes into play when a body just start moving over the surface of another body is called limiting friction

Clearly, $f_s \leq (f_s^{\max})$

Kinetic friction :-

The force of friction which comes into play when a body is a state of steady motion over the surface of another body is called kinetic friction (f_k)

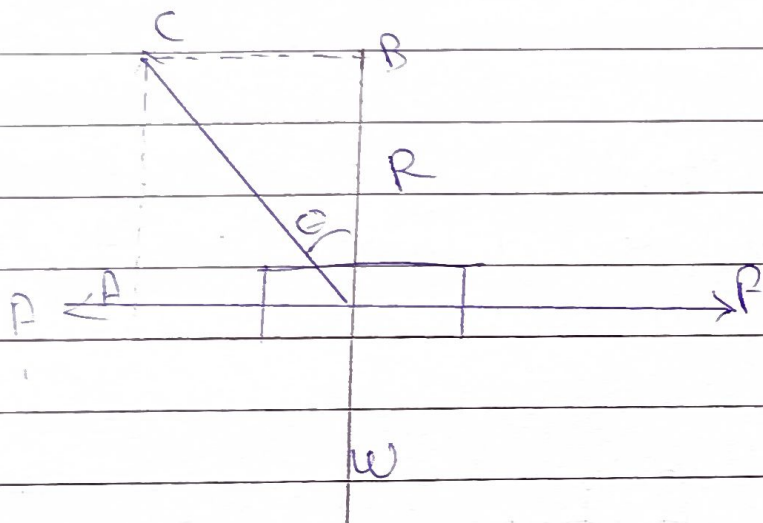
Variation of the force of friction with the applied force =



Angle of friction

It is defined as the angle which the resultant of the limiting friction and the normal reaction makes with the normal reaction.

Relation between angle of friction and Coefficient of friction



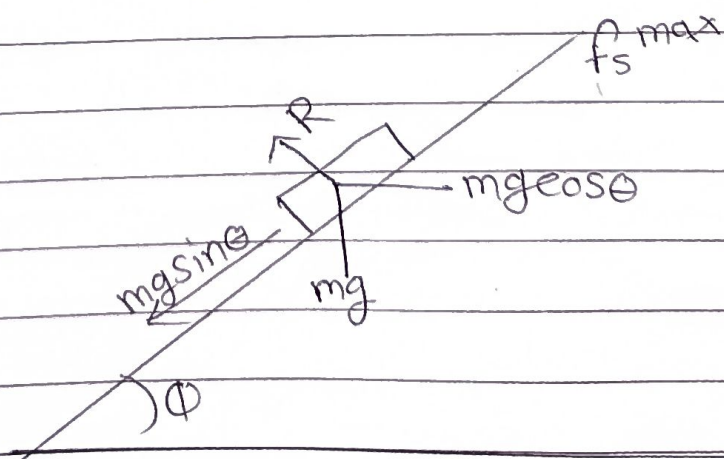
$$\tan \theta = \frac{f_s^{\max}}{R} = \mu_s$$

$$\therefore \tan \theta = \mu_s$$

"The coefficient of static friction is equal to the tangent of angle of friction".

Angle of Repose :-

It is the maximum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down.



At equilibrium,

$$mg \sin \phi = f_s \text{ max} \quad \text{--- (1)}$$

$$mg \cos \phi = R \quad \text{--- (2)}$$

From eq (1) and (2)

$$\frac{\sin \phi}{\cos \phi} = \frac{f_s \text{ max}}{R}$$

$$\tan \phi = \frac{f_s \text{ max}}{R} = \mu_s$$

$$\boxed{\tan \phi = \mu_s}$$

"The coefficient of static friction is equal to the tangent of the angle of repose."

$$\boxed{\tan \phi = \mu_s}$$

$$\therefore \phi = \theta$$

Angle of repose = Angle of friction

Types of kinetic friction

- 1) Sliding friction
- 2) Rolling friction

Sliding friction :

The force of friction that comes into play when a body slides over the surface of another body is called sliding friction

Rolling friction :

The force of friction, that comes into play when a body rolls over the surface of another body is called rolling friction

Advantage of friction

- 1) The brakes of a vehicle can't work without friction.
- 2) It is due to friction between the ground and the feet that we are able to walk
- 3) In the absence of friction it will not be possible to write on a paper with a pencil or pen

Disadvantage of friction

- 1) Wear and tear of the machinery is due to friction.
- 2) A large amount of power is wasted.

Method of Reducing friction

- 1) By Lubrication
- 2) By polishing
- 3) By using ball bearing

Centripetal force:

A force required to make a body moves along a circular path with uniform speed is called centripetal force. It always act along the radius and

$$F_c = ma_c$$

$$F_c = \frac{mv^2}{r}$$

$$F_c = m\omega^2 r$$

Circular motion of a car on a level road:

- The total force of friction is equal to $f_1 + f_2 = \mu f = \mu mg$, for the car, to stay on the road the max. force of friction must be equal to or greater than the centripetal force

$$\mu mg \geq \frac{mv^2}{r}$$

$$\mu mg \geq v^2 = \mu rg > v^2$$

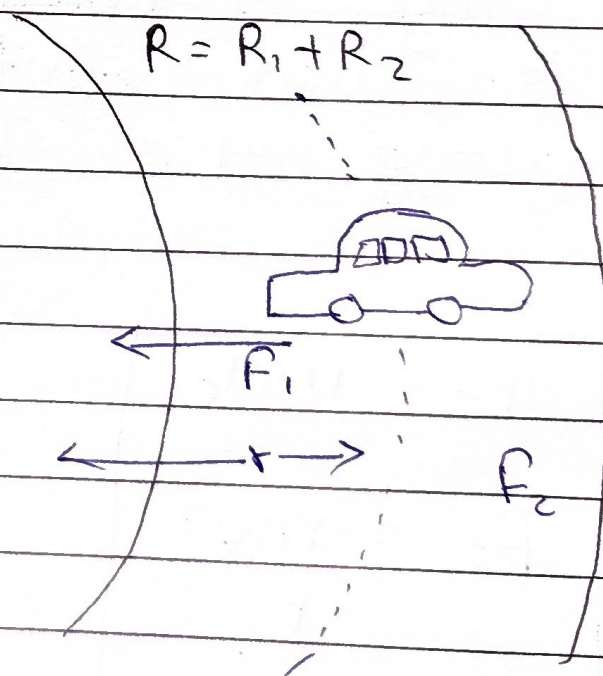
$$= v^2 \leq \mu rg$$

$$= v < \sqrt{\mu rg}$$

The max. Speed with which the car can turn safely

$$V_{\max} = \sqrt{\mu rg}$$

for skidding $\rightarrow V_{\max} \rightarrow \sqrt{\mu rg}$



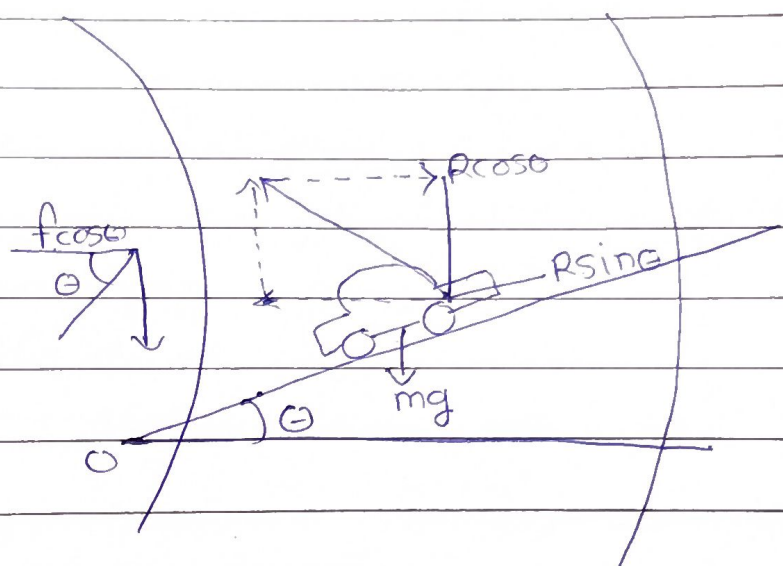
Circular motion of a car on a bank road

At equilibrium :-

$$f \cos \theta + R \sin \theta = \frac{mv^2}{r} \quad \text{--- (1)}$$

$$f \sin \theta + mg = R \cos \theta$$

$$R \cos \theta = f \sin \theta + mg \quad \text{--- (2)}$$



Dividing eq (1) and (2)

$$\frac{f \cos \theta + R \sin \theta}{R \cos \theta - f \sin \theta} = \frac{v^2}{rg}$$

$$\frac{f \frac{\cos \theta}{\cos \theta} + R \frac{\sin \theta}{\cos \theta}}{R \frac{\cos \theta}{\cos \theta} - f \frac{\sin \theta}{\cos \theta}} = \frac{v^2}{rg}$$

$$\frac{f + R \tan \theta}{R - f \tan \theta} = \frac{v^2}{rg}$$

$$v^2 = rg \left(\frac{f + R \tan \theta}{R - \mu R \tan \theta} \right)$$

$$v = \sqrt{rg \left(\frac{\mu R + R \tan \theta}{R - \mu R \tan \theta} \right)}$$

$$v = \sqrt{rg \left(\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right)}$$

This is the required velocity of the car at the banked road

when there is no friction between the road and tire, the $\mu = 0$

$$v = \sqrt{rg \tan \theta}$$

The angle of banking θ for min. wear & tear of tires :-

$$v^2 = rg \tan \theta$$

$$\tan \theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$